(Pages : 4) **R – 1232**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2023

First Degree Programme under CBCSS

Mathematics

Core Course XIII

MM 1645 : INTEGRAL TRANSFORMS

(2018 Admission onwards)

Time : 3 Hours Max. Marks : 80

PART – A

All the first ten questions are compulsory. They carry **1** mark each.

1. Find the Laplace transform of $f(t)=e^{-3t}$.

2. If $L[f(t)]=F(s)$, then $L\{\int_0 t f(u)du\}=$ ————

3. Find the inverse Laplace transform of $\frac{1}{s^2+4}$.

4. Define unit step function.

5. Write $L\{f''(t)\}$ in terms of $L(f)$, $f(0)$ and $f'(0)$.

6. Give an example of a periodic function which is neither odd nor even.

P.T.O.

7. Define Fourier cosine transform of a function $f(t)$.

8. What is the standard form of Fourier series for an even function?

9. Find the fundamental period of the function $\cos(nx)$.

10. If $f(x)$ has period p then find the period of $f(2x)$.

(10 × 1 = 10 Marks)

PART – B

Answer **any eight** questions. Each question carries **2** marks.

11. Find the Laplace transform of $f(t) = t \sin 2t$.

12. Find $L[e^{-3t} \sin 3t]$.

13. Evaluate $L^{-1}\left[\frac{3}{(s+2)^4}\right]$.

14. Find $L^{-1}\left(\frac{1}{(s+2)(s+3)}\right)$.

15. Solve the differential equation $y'' - y = t$, $y(0) = 1$, $y'(0) = 1$.

16. Define convolution of two functions and find the convolution of 1 and -1.

17. Is $L[f(t) + g(t)] = L(f(t)) + L[g(t)]$? Explain.

18. Find the Fourier series of $f(x) = x$ for $0 < x < 2\pi$.

19. Find the Fourier sine series for the function $f(x) = \pi - x$ in $0 < x < \pi$.

20. Find the Fourier cosine transform of the function $f(x) = \begin{cases} k, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$.

21. Derive the Fourier transform of $f'(x)$, the derivative of $f(x)$.

22. Check whether the following functions are odd or even

(a) $\frac{x}{x^2+1}$

(b) $x^3 \tan \pi x$.

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions. Each question carries **4** marks.

23. Find the Laplace transform of the function $f(t)=\begin{cases} t & t \geq 3 \\ 0 & t<3 \end{cases}$.

24. Find the inverse transform of $(3s+1)/(s-1)\left(s^2+1\right)$.

25. Solve the equation $y''+3y'+2y=\delta(t-1), \; y(0)=0, y'(0)=0$.

26. Find the Laplace transform of $e^{-4t} \int_0^t t \sin 3t \, dt$.

27. Find the Laplace transform of $t^2 e^{-3t} \sin 2t$.

28. Find the inverse Laplace transform of $\frac{2\left(e^{-s}-e^{-3s}\right)}{s^2-4}$.

29. Find the inverse of $\frac{5s+3}{(s-1)\left(s^2+2s+5\right)}$.

30. Find the Fourier series for $f(x)=|x|$ from $x=-\pi<x<\pi$ and deduce that

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots=\frac{\pi^2}{8}.$$

31. Using Fourier sine integral, Show that

$$\int_0^\infty \frac{1-\cos\pi w}{w}\sin xw\,dw=\begin{cases}\frac{\pi}{2} & \text{if } 0<x<\pi\\ 0 & \text{if } x>\pi\end{cases}$$

(6 × 4 = 24 Marks)

PART – D

Answer **any two** questions. Each question carries **15** marks.

32. (a) State and prove convolution theorem of Laplace transforms

(b) Use convolution theorem to find the inverse Laplace transform of $\frac{1}{s^2(s-a)}$.

33. Find the Fourier series of the periodic function $f(x)$ of period 2, where $f(x)=\pi x$ in $0\leq x\leq 2$ and deduce that $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots=\frac{\pi}{4}$.

34. (a) Represent $f(x)=\begin{cases}\sin x, & 0<x<\pi\\ 0, & x>\pi\end{cases}$ as Fourier cosine integral

(b) Find the Fourier integral of $f(x)=\begin{cases}\pi-x & \text{if } 0<x<\pi\\ 0, & \text{otherwise}\end{cases}$

35. (a) Find the Fourier transform of $f(x)=e^{-\frac{x^2}{2}}$. What is your inference?

(b) Find the Fourier transform of $f(x)=\begin{cases}|x| & \text{if } -1<x<1\\ 0, & \text{otherwise}\end{cases}$

(2 × 15 = 30 Marks)

(Pages : 4) **R – 1231**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2023

First Degree Programme under CBCSS

Mathematics

Core Course XII

MM 1644 : LINEAR ALGEBRA

(2018 Admission onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Write the given system of equations in column form
   $5x + 20y = 80$
   $-x + 4y = -64$
2. Find two points on the line of intersection of the three planes $t = 0, z = 0$ and $x + y + z + t = 1$ in four dimensional space.
3. Define a symmetric matrix.
4. Define subspace of a vector space.
5. The columns of a matrix A and n vectors from R^m. If they are linearly independent, what is the rank of A?
6. Write the reflexion matrix that transforms (x, y) to (y, x).
7. If a 3 by 3 matrix has $\det A = \frac{-1}{2}$, find $\det(A^{-1})$.

P.T.O.

8. State whether true or false: The determinant of $S^{-1}AS$ equals the determinant of A.

9. If $\lambda \neq 0$ is an eigen value of A and x is its eigen vector, then show that x is also an eigen vector of A^{-1}.

10. Define a transition matrix.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. Write down the upper triangular system of equations. Also find the pivots

$$2x + 3y = 1$$
$$10x + 9y = 11$$

12. Give 2 by 2 matrices A and B such that AB = 0, although no entries of A or B are zero.

13. Describe the null space of $A = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$.

14. Describe the subspace of $\mathbb{R}^3$ spanned by the two vectors (1, 1, –1) and (–1, –1,1).

15. Check whether the transformation $T : R^3 \to R^3$ defined by $T(V_1, V_2, V_3)$ = $(V_2, V_3, 0)$ is linear.

16. Using the big formula, compute det A from 6 terms. Are rows independent?

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 4 & 4 \\ 5 & 6 & 7 \end{bmatrix}$$

17. The corners of a triangle are (2,1), (3,4) and (0,5). What is the area?

18. Suppose the permutation P takes (1,2,3,4,5) to (5,4,1,2,3). What does P^{-1} do to (1,2,3,4,5).

19. Show that the eigen value of A equals the eigen value of A^T.

20. Factor $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ into $S \Lambda S^{-1}$.

21. Compute A^3 for the Fibonacci matrix $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$.

22. Prove that two eigen vectors of a Hermitian matrix, if they come from different eigen values one orthogonal to one another.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

23. For which numbers 'a' does the elimination break down (i) Permanently (ii) Temporarily $\begin{matrix} ax+3y=-3 \\ 4x+6y=6 \end{matrix}$

Solve for x and y after fixing the second break down by a row exchange.

24. Find all matrices $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ that satisfy $A=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}A$.

25. Reduce to echelon form $\begin{bmatrix} 0 & 0 & 2 \\ 1 & -1 & 1 \\ -1 & 1 & -4 \end{bmatrix}$.

26. Find the range and kernel of $T(V_1,V_2)=(V_1 V_1+V_2)$.

27. Find x, y and z by Cramer's rule :

$2x+y=1$

$x+2y+z=70$

$y+2z=0$

28. Define orthogonal matrix. Prove that if A is an orthogonal matrix, then $\det A=\pm 1$.

29. Find the eigen values and eigen vectors of $A=\begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$.

30. If $A=\begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$, find A^{100} by diagonalising it.

31. Compute $A^H A$ and AA^H for $A=\begin{bmatrix} i & 1 & i \\ 1 & i & i \end{bmatrix}$.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

32. (a) Compute the symmetric LDL^T factorisation of $A=\begin{bmatrix}1 & 3 & 5\\ 3 & 12 & 18\\ 5 & 18 & 30\end{bmatrix}$.

(b) Use the Gauss-Jordan method to invert $\begin{bmatrix}1 & 2 & 1\\ 3 & -2 & -4\\ 2 & 3 & -1\end{bmatrix}$.

33. (a) Find the value of α that makes the system solvable and find the solution

$x+y+z=1$

$2x-y+2z=1$

$x+2y+z=\alpha$

(b) Find the dimensions of the column space and raw space of $A=\begin{bmatrix}1 & 1 & 0\\ 0 & 2 & 1\\ 0 & 0 & 0\end{bmatrix}$.

34. (a) Find the dimension and a basis for the four fundamental subspaces for $u=\begin{bmatrix}1 & 2 & 0 & 1\\ 0 & 1 & 1 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$

(b) Find the 4 by 4 matrix A that represents a right shift : (x_1, x_2, x_3) is transformed to $(0, x_1, x_2, x_3)$. Also find the left shift matrix B from R^4 back to R^3, transforming (x_1, x_2, x_3, x_4) to (x_2, x_3, x_4).

35. (a) Test the cayley-Hamilton theorem on $A=\begin{bmatrix}1 & 2\\ 4 & 3\end{bmatrix}$.

(b) Find a diagonal matrix M so that $A=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$ is similar to $\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$.

(2 × 15 = 30 Marks)

(Pages : 4) **R – 1228**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2023

First Degree Programme under CBCSS

Mathematics

Core Course XI

MM 1643 : ABSTRACT ALGEBRA-RING THEORY

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. Each carries **1** mark.

1. Give an example of commutative ring with zero divisors.

2. Which are units of Z_5?

3. What is the characteristic of nZ?

4. Find the number of elements in the factor ring $\frac{2z}{8z}$.

5. State factor theorem.

6. Is the ring $2z$ isomorphic to the ring $4z$?

7. Define a primitive polynomial.

P.T.O.

8. Define a unique factorisation domain.

9. Find the norm of $1+\sqrt{-5}$.

10. State whether true or false: Every Euclidean domain is a UFD.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. Each carries **2** marks.

11. Prove that $a(-b)=(-a)b=-(ab)$ in a ring R.

12. Show that if a and b are idempotents in a commutative ring, then ab is also idempotent.

13. Prove that the ideal $< x^2+1 >$ is not prime in $z_2[x]$.

14. Define a principal ideal domain. Give an example.

15. Find the kernel of the ring homomorphism from $R[x]$ to R defined by $f(x) \to f(1)$.

16. Is the field of real numbers is ring isomorphic to the field of complex numbers? Justify your answer.

17. Show that the polynomial $2x+1$ in $Z_4[x]$ has a multiplicative inverse in $Z_4[x]$.

18. Construct a field of 9 elements.

19. Prove that every subring of Z is of the form nZ for some $n \in Z$.

20. Define associates and irreducibles in an integral domain.

21. Suppose that a and b belong to an integral domain, $b \neq 0$ and a is not a unit. Show that $< ab >$ is a proper subset of $< b >$.

22. Let $d < -1$ be an integer, that is not divisible by the square of a prime. Prove that the only units of $Z(\sqrt{d})$ are ± 1.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. Each question carries **4** marks.

23. Prove that the set $\left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} | a, b \in Z \right\}$ is a subring of the ring of all 2 × 2 matrices over Z.

24. Find all solutions of the equation $x^2 - 5x + 6 = 0$ in Z_{12}.

25. Show that an intersection of subfields of a field F is again a sub field of F.

26. Let φ be a ring homomorphism from a ring R to a ring S. If B is an ideal in S, then prove that $\phi^{-1}(B) = \{r \in R \mid \phi(r) \in B\}$ is an ideal of R.

27. Let R be a ring with unity 1. Then show that the mapping $\phi : Z \to R$ by $\phi(m) = m.1$ is a ring homomorphism.

28. Show that if D is an integral domain, then $D[x]$ is also an integral domain.

29. Show that $1 + \sqrt{-3}$ is irreducible in $Z[\sqrt{-3}]$.

30. Show that the ring of Gaussian integers $Z[i] = \{a + bi \mid a, b \in z\}$ is a Euclidean domain.

31. In $Z[\sqrt{-5}]$, show that 21 doesn't factor uniquely as a product of irreducibles.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** carries **15** marks.

32. (a) Prove that a finite integral domain is a field.

    (b) Prove that the characteristic of an integral domain is zero or prime.

33. Let D be an integral domain. Then prove that there exist a field F that contains a subring isomorphic to D.

34. (a) State and prove Gauss's lemma.

(b) Let $f(x) \in Z[x]$. Prove that if $f(x)$ is reducible over Q, then it is reducible over Z.

35. Prove that in a PID, an element is irreducible if and only if it is prime.

(2 × 15 = 30 Marks)

(Pages : 4) **R – 1225**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2023

First Degree Programme under CBCSS

Mathematics

Core Course – X

MM 1642 : COMPLEX ANALYSIS – II

(2018 Admission onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Define convergence of series of complex numbers.

2. Define Uniform Convergence of a sequence of functions.

3. Define a power series.

4. Define a simple pole.

5. Define residue of $f(z)$ at z_0.

6. Define Improper integral over $[0,\infty]$ of a continuous non negative function $f(x)$.

7. Define the Cauchy principal value of f over $(-\infty,\infty)$

8. What do you mean by local property of a mapping.

P.T.O.

9. Define open mapping property.

10. Define the mapping: Translation

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. Prove that $\sum_{j=1}^{\infty} C^{j}$ converges to $\frac{1}{1-c}$ for $|c|<1$.

12. Find the Maclaurin series for $\sin z$.

13. Define Cauchy Product of two Taylor series.

14. State M test for uniform convergence.

15. State the necessary and sufficient condition for an analytic function to have a zero of order m at z_0.

16. Define the radius on convergence of a power series.

17. If $f(z)=\frac{P(z)}{Q(z)}$, where $P(z)$ and $Q(z)$ are analytical at z_0 and Q has a simple pole at z_0, while $P(z_0)\neq 0$, derive the formula for $\text{Res}(f;z_0)$

18. Find the residue at $z=0$ of $f(z)=z^2\sin\left(\frac{1}{z}\right)$ using Laurent series.

19. Evaluate $p.v.\int_{-\infty}^{\infty} x\,dx$.

20. Show that e^z is locally one to one but not globally.

21. State the Riemann Mapping Theorem.

22. Define the mappings: Rotation and Magnification.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

23. State the comparison test and show that $\sum_{j=1}^{\infty} \frac{3+2i}{(j+1)^j}$ converges.

24. Find the Taylor series of $Log\, z$ around $z = 1$.

25. Prove that the uniform limit of a sequence of continuous functions defined on a simply connected domain is also continuous.

26. Classify the zeros and singularities of $\sin\left(1 - \frac{1}{z}\right)$.

27. Find the residues at each singularity of $f(z) = \cot z$.

28. Prove that, if $f(z)$ has a pole of order m at z_0, then

$$\operatorname{Re} s(f : z_0) = \lim_{z \to z_0} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} [(z - z_0)^m f(z)].$$

29. Suppose that $f(t)$ and $M(t)$ are continuous function defined on $[a, b]$, with f complex and M real valued. Prove that if $|f(t)| \leq M(t)$ on this interval, then $\left|\int_a^b f(t)\,dt\right| \leq \int_a^b M(t)\,dt$.

30. Prove that if $f(z)$ is analytic at z_0, and $f'(z_0) \neq 0$ then $f(z)$ is conformal at z_0.

31. Define a Mobius transformation and show that the inverse of a Mobius transformation is another Mobius Transformation

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

32. Prove that if $f(z)$ is analytic in the disk $|z - z_0| < R$, then there exists a Taylor series which converges to $f(z)$ for all z in this disk.

33. (a) Expand $e^{(1/z)}$ as a Laurent series about $z = 0$.

    (b) Find the Laurent Series for $f(z) = \dfrac{z^2 - 2z + 3}{z - 2}$ in the region $|z - 1| > 1$.

34. State and prove Cauchy Residue Theorem. Using Cauchy Residue Theorem evaluate the integral of $f(z) = \dfrac{1 - 2z}{z(z-1)(z-2)}$ over the circle $|z| = 2$.

35. Evaluate $\displaystyle\int_0^{2\pi} \frac{\sin^2\theta}{5 + 4\cos\theta}\, d\theta$.

(2 × 15 = 30 Marks)

(Pages : 4) **R – 1222**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2023

First Degree Programme under CBCSS

Mathematics

Core Course IX

MM 1641 : REAL ANALYSIS – II

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Evaluate $\lim_{x \to 7/4} \frac{|x-2|}{x-2}$

2. State true or false: Every uniformly continuous function is continuous.

3. Determine the points of discontinuity of the greatest integer function.

4. State the mean value theorem.

5. Define a uniformly continuous function.

6. Define a differentiable function at a point.

7. Give an example of a real valued function which is discontinuous at every point of R.

8. Define upper integral of a function f.

9. When do you say that a bounded real function f is integrable on [a,b]?

10. State true or false: If $|f|$ is integrable on [a,b] then f is also integrable on [a,b].

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. Each question carries **2** marks.

11. Evaluate $\lim_{x \to 0} \frac{x^2}{|x|}$

12. Prove that the Dirichlet's function f defined on R by $f(x) = \begin{cases} 1 \text{ if } x \text{ is irrrational} \\ -1 \text{ if } x \text{ is rational} \end{cases}$

is discontinuous at every point.

13. If f: $A \to$ R and g: $A \to R$ are continuous at a point $c \in A$, show that $f(x)+g(x)$ is also continuous at c.

14. Is the function $f(x) = \frac{1}{x}$ uniformly continuous on (0, 1]? Justify.

15. Prove that $\{f(x_n)\}$ is a Cauchy sequence for every Cauchy sequence $\{x_n\}$ in R where f is a uniformly continuous function.

16. If f is differentiable in (a, b) and $f'(x) \le 0$ for all $x \in (a,b)$, show that f is monotonically decreasing.

17. Show by an example that a bounded function in [a,b] need not be continuous in [a,b].

18. If f: $A \to R$ is differentiable at a point $c \in A$, then f is continuous at c as well.

19. Find the value of δ for the function $f(x) = x^2 + 4x + 3$ to be uniformly continuous in the interval [-1,1], given $\varepsilon = \frac{1}{10}$.

20. Check whether the following function is integrable over [0,1]: $f(x) = 1$ if x $\in$ [0,1] and x is rational and $f(x) = 0$ if $x \in [0,1]$ and x is irrational.

21. Show that $\underline{\int_a^b} f\,dx \geq \int_a^{\overline{b}} f\,dx$.

22. Show that if f and g are bounded and integrable on [a,b], such that $f \geq g$, then $\int_a^b f\,dx \geq \int_a^b g\,dx$.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. Each question carries **4** marks.

23. Test the continuity of the function at x = 0 $f(x) = \begin{cases} x \sin\frac{1}{x}, for\ x \neq 0 \\ 0, for\ x = 0 \end{cases}$

24. Explain Lipschitz functions with the geometrical interpretation.

25. Show that a uniformly continuous function preserves Cauchy sequences.

26. Suppose f is a real differentiable function on [a,b] and suppose $f'(a) < \lambda < f'(b)$. Prove that there is a point $x \in (a,b)$ such that $f'(x) = \lambda$.

27. State and prove chain rule of differentiation.

28. State and prove Darboux's theorem.

29. Prove that, if f is monotonic in [a,b], it is integrable in [a,b].

30. If f and g are integrable in [a,b] then show that fg is also integrable in [a,b].

31. Show that the Dirichlet's function $g(x) = \begin{cases} 1\, for\ x\ rational \\ 0\, for\ x\ irrational \end{cases}$ is not integrable.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. Each question carries **15** marks.

32. Let f: $A \to R$ be continuous on A. If $K \subseteq A$ is compact, then prove that f (K) is compact as well.

33. State and prove Intermediate value theorem. Is the converse true? Justify your answer.

34. Prove that a bounded function f is integrable on [a,b] if and only if for every $\varepsilon > 0$ there exists a partition P such that U (P,f) – L (P, f) < ε.

35. If f is bounded and integrable on [a,b] and k is a number such that $|f(x)| \le k$ for all x $\in [a, b]$. Prove that $\left|\int_a^b f\, dx\right| \le k(b-a)$.

(2 × 15 = 30 Marks)